Graph Theory

By

Dr. Neena Gupta

Dean, Training, Career Group of Institutions

Assistant Professor, Mathematics

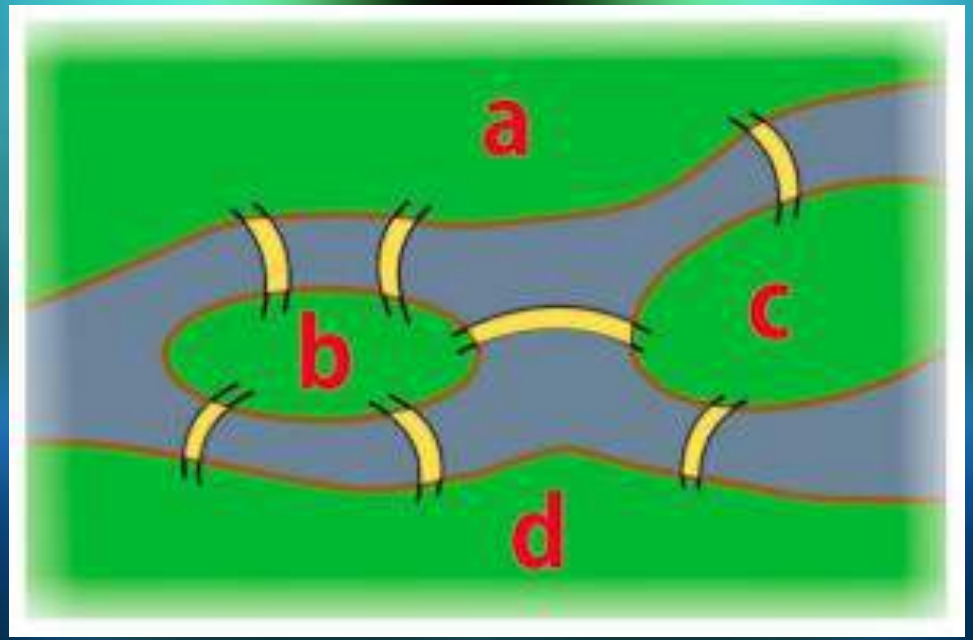
Career College, Bhopal

Email: gneena33@gmail.com

Course: B. Sc.- III Year

Paper: M III - Discrete Mathematics

Unit: V



Learning Objectives

To understand and apply the fundamental concepts in graph theory

To apply graph theory-based tools in solving practical problems

To improve the proof writing skills

Learning Outcomes

Students will be able to

- Apply principles and concepts of graph theory in practical situations
- Solve problems using basic graph theory
- Identify induced subgraphs, cliques, matchings, covers in graphs

Outline Of Graph Theory

- Introduction of Graph Theory
- Fundamentals of Graph
- Types of Graph
- Isomorphic Graph
- Summary
- Reference
- Self Assessment

Introduction

FATHER OF GRAPH THEORY

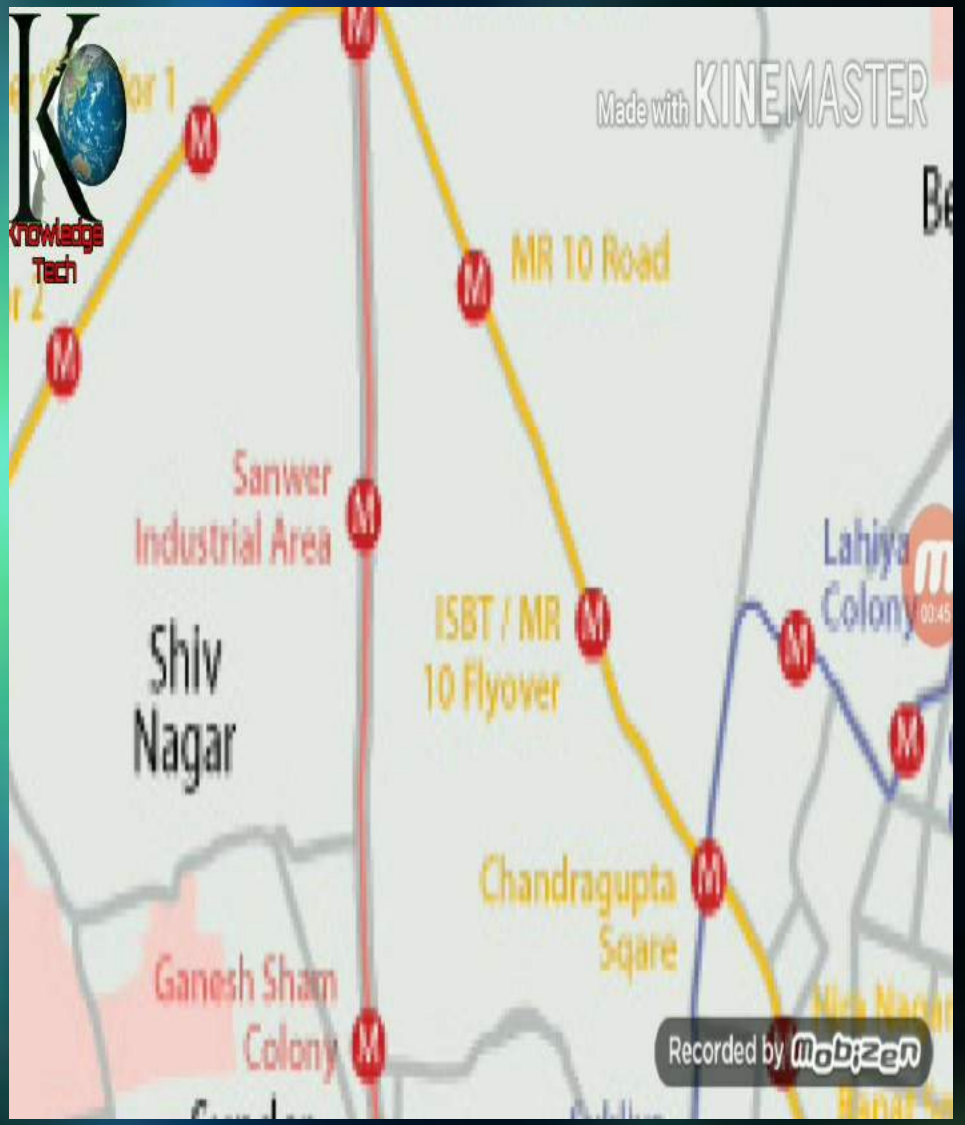
The great Swiss Mathematician

Leonhard Euler

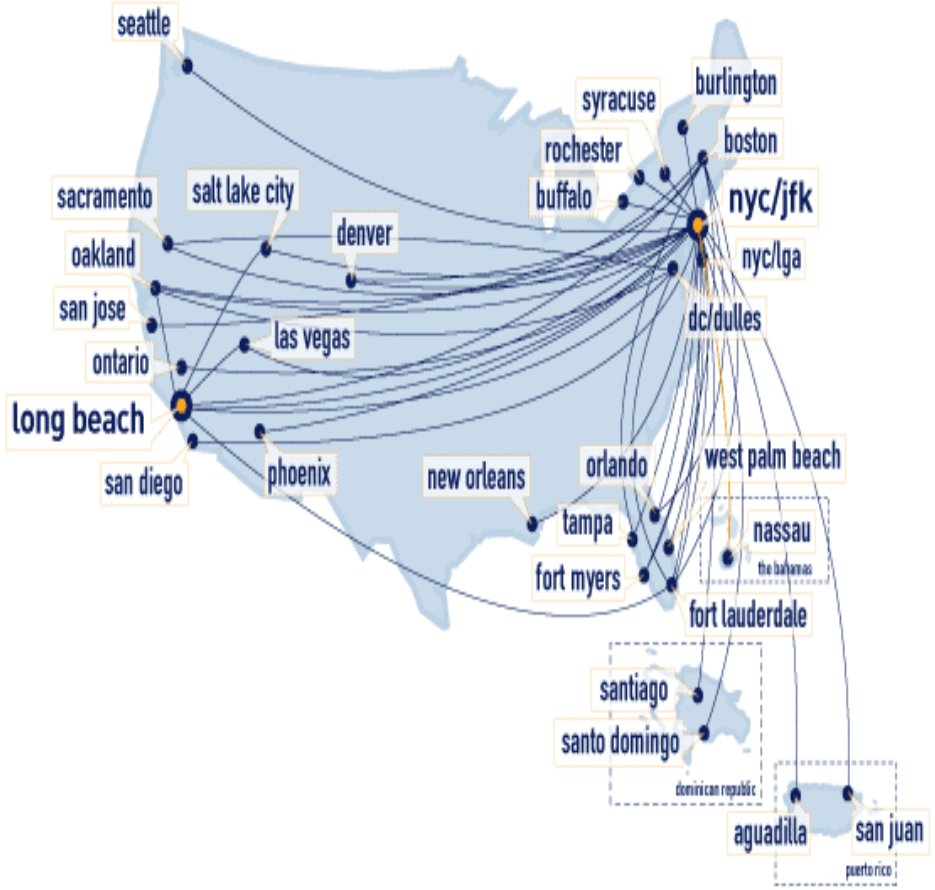
Introduction

What is a Graph

- A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links.
- The interconnected objects are represented by points termed as vertices. The links that connect the vertices are called edges.
- A graph is a pair of sets (V, E), where V is the set of vertices and E is the set of edges, connecting the pairs of vertices.



Application of Graph

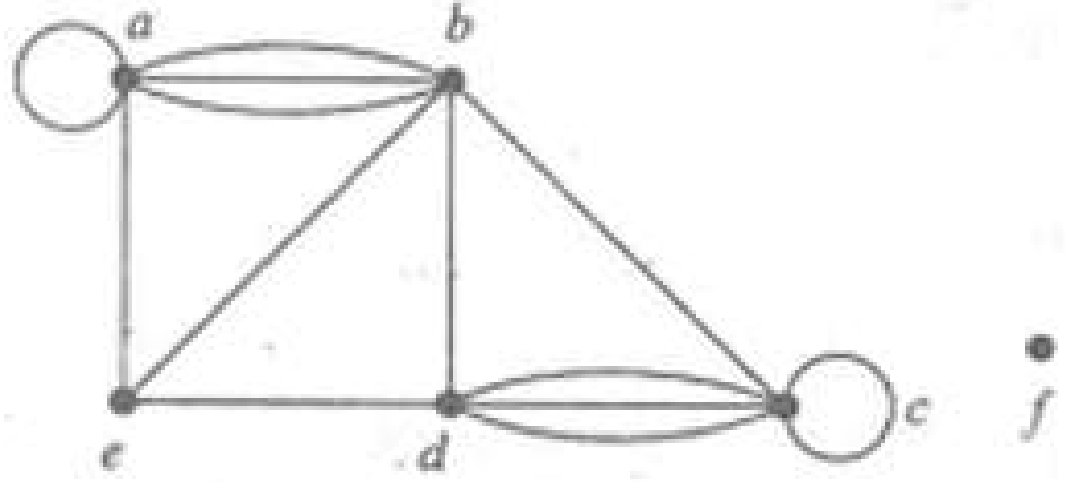


- Graph theory has proven useful in the design of Integrated circuits (ICs) for computers and other electronic devices.
- Graph plays a major role in each and every branch
- Graph theory has its applications in diverse fields of engineering –**Electrical Engineering, Computer Science, Computer Network, Science, Linguistics, general**.

Preliminaries

Self Loop

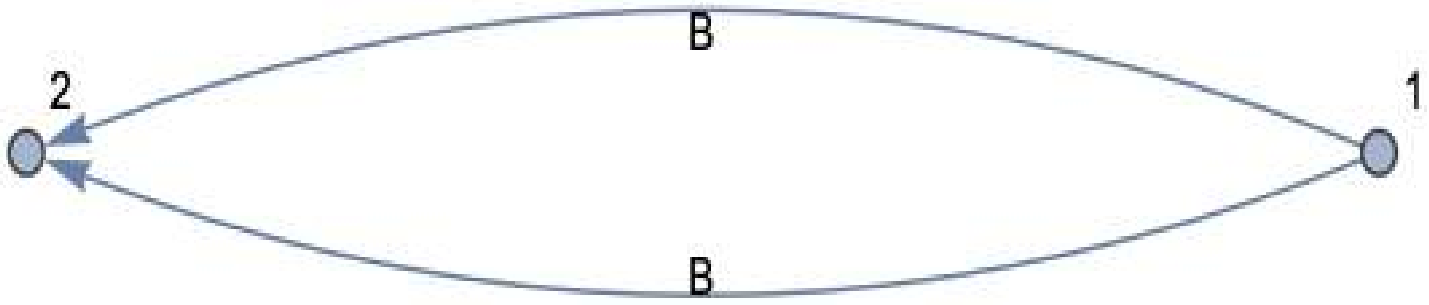
- Edge having the same vertex as both its end vertices (initial vertex and terminal vertex).
- Form of Edge of self loop : (v_i, v_i).
- In this below mentioned example vertex a and c contains an edge known as Self loop; whose starting or initial vertex and terminal or ending vertex are same.



Parallel Edges

- Edges in between any two vertices : Considered as Parallel edges.

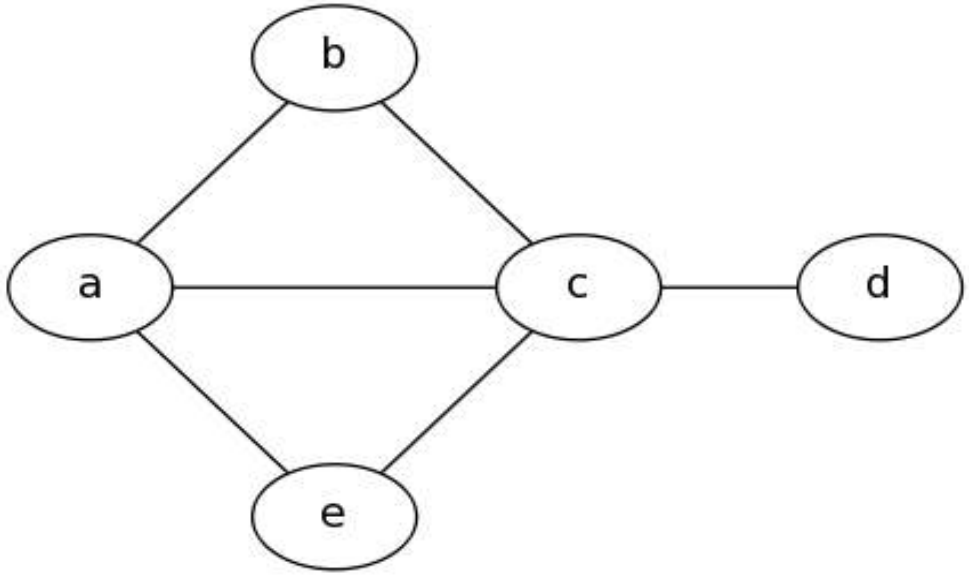
- In this mentioned example there are two edges in between the same initial vertex 1 and the terminal vertex 2 known as parallel edges.



Simple Graph

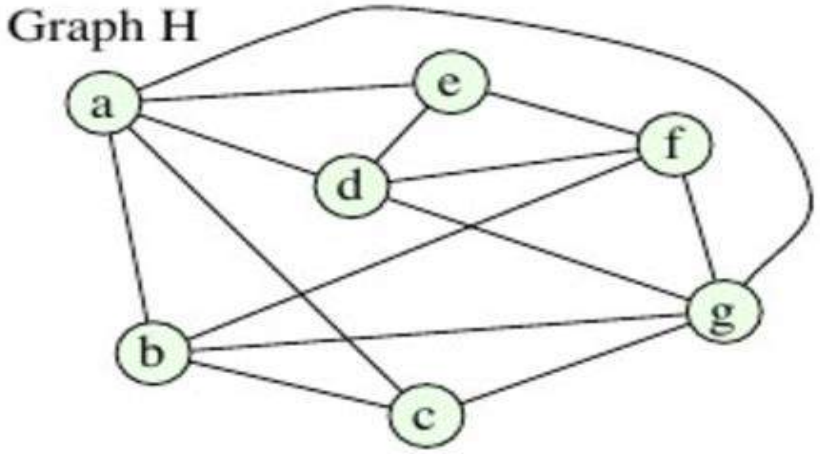
- Graph G=(V,E); which has neither self loop nor parallel edges.

- In this following example, there is no such vertex which can be called as both initial vertex and terminating vertex, i.e., no self loop and also there are no two edges in between the same initial and terminal vertex, i.e., no parallel edges.



ADJACENT VERTEX AND ADJACENT EDGES

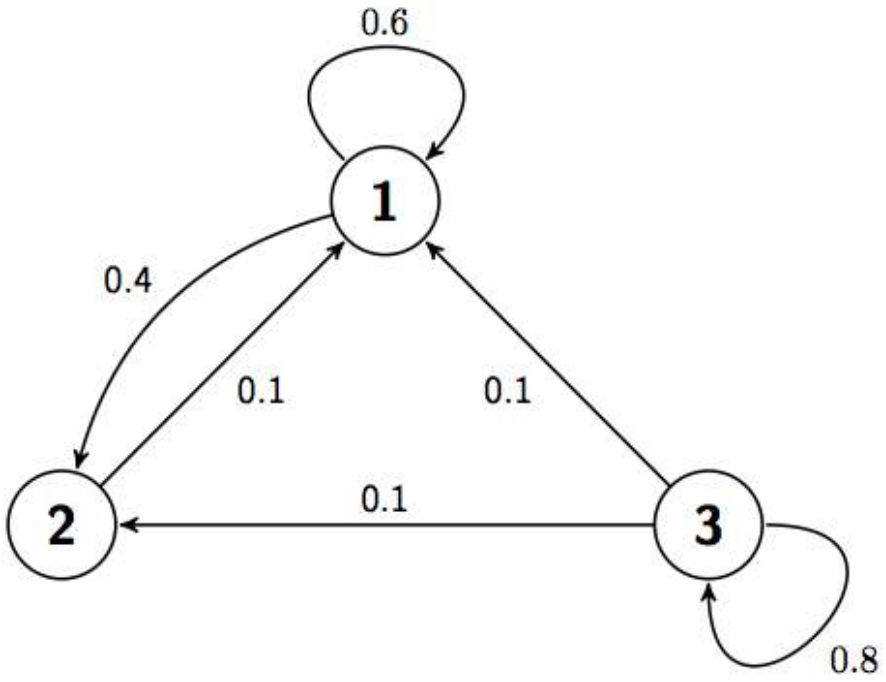
- Two vertex are said to be adjacent if they share a common edge.
- Two non parallel edges are said to be adjacent if they are incident on a common vertex.
- In this example a, b, c, d, e, f, g are the elements of set V known as vertices and connection in between these vertices is known as edge. Vertex a is adjacent to b, e and d; a is not adjacdnt to f, g and c. Edges (a,e) and (a,b) are Adjacent edges as they incident on a common vertex a.



DEGREE OF VERTEX

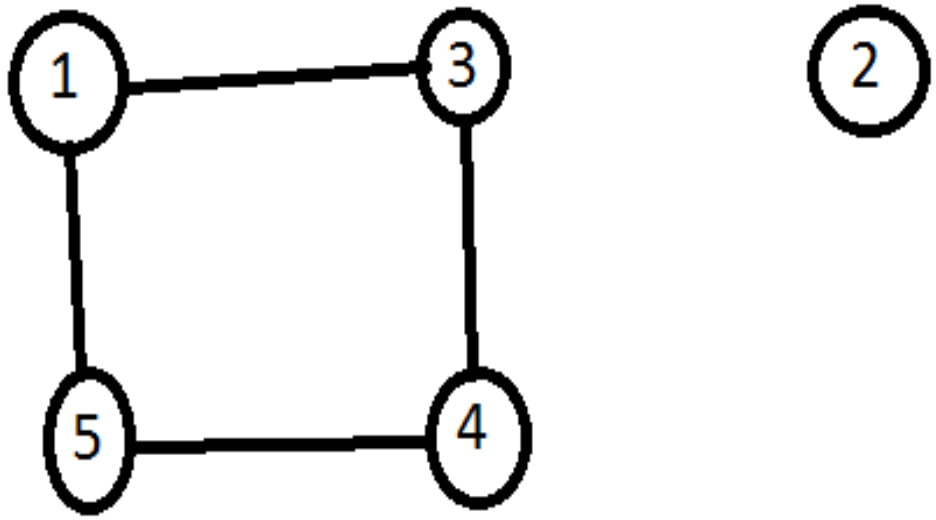
- It is the number of edges incident on any vertex with self loop counted as twice.

- In this following mentioned example:
  Degree of vertex 1 = d(1)=5; Similarly, d(2) = 3, d(3) = 4.



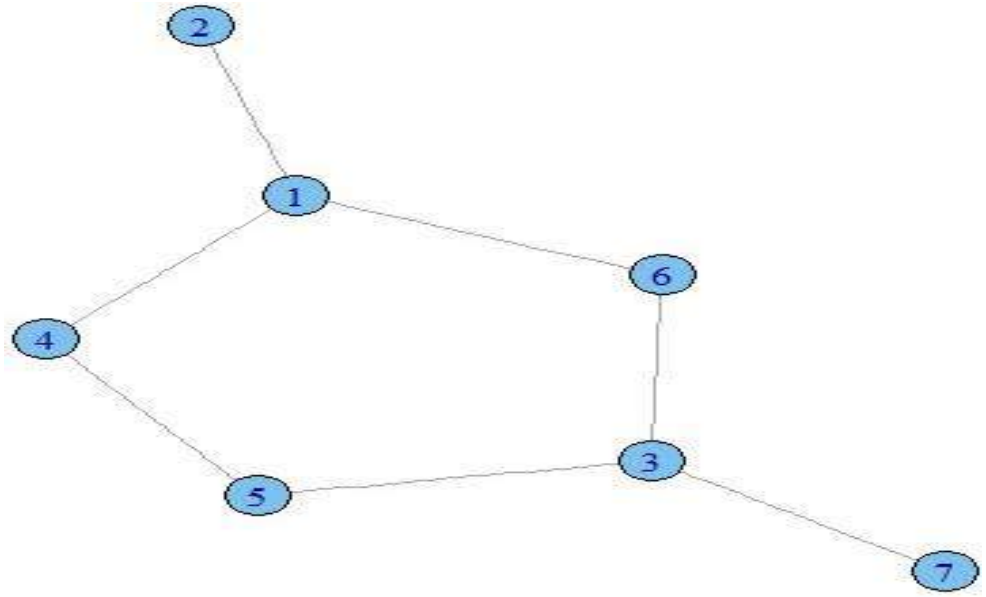
Isolated Vertex

- A vertex in a graph G is said to be isolated, if no edge incident on it.
- Degree of isolated vertex = 0 (as no edge incident on it).
- In the following example vertex 2 is an isolated vertex as no edge incident on it; due to which its degree is 0.



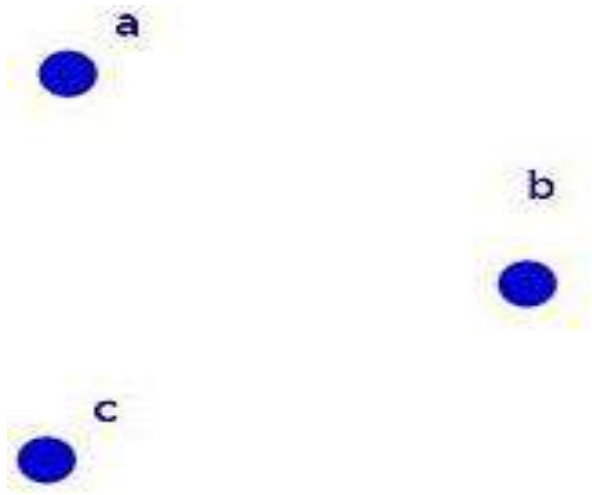
Pendent Vertex

- A vertex in a graph G, is said to be pendant vertex if only one edge incident on it.
- Degree of Pendant Vertex = 1.
- In this mentioned example of graph, vertex 2 and vertex 7 is a pendant whose degree is 1.



Null Graph

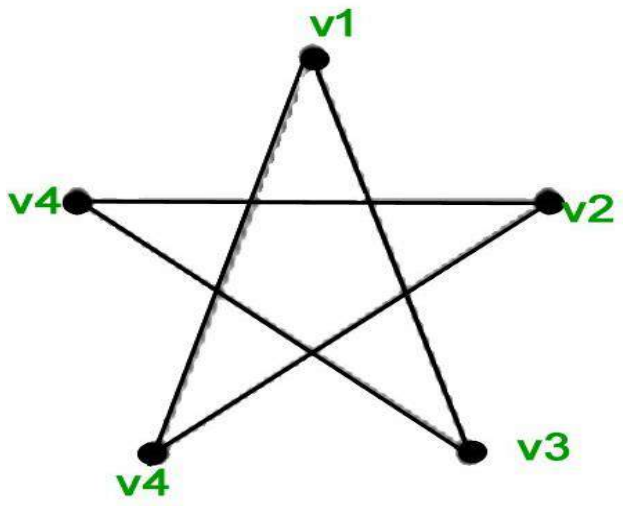
- Collection of isolated vertices known as Null Graph.
- Degree of each and every vertex of a Null Graph = 0.
- In this mentioned graph, Vertex a, b and c are isolated vertices that's why it is known as Null Graph.

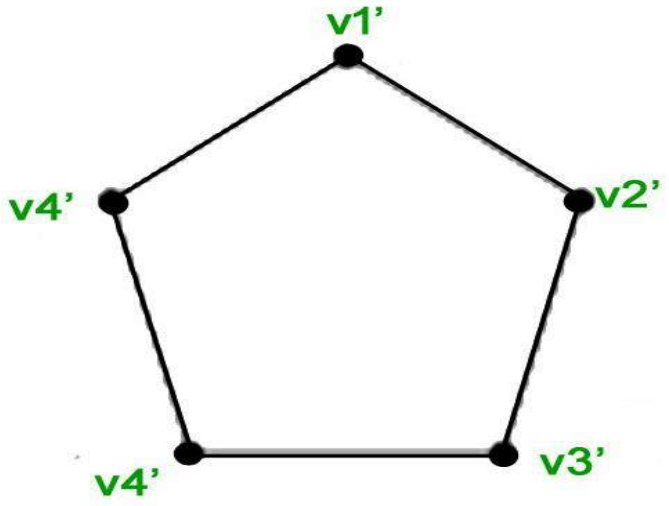


Isomorphic Graph

- If degree of any two graphs are equal then it said to be an Isomorphic Graph.

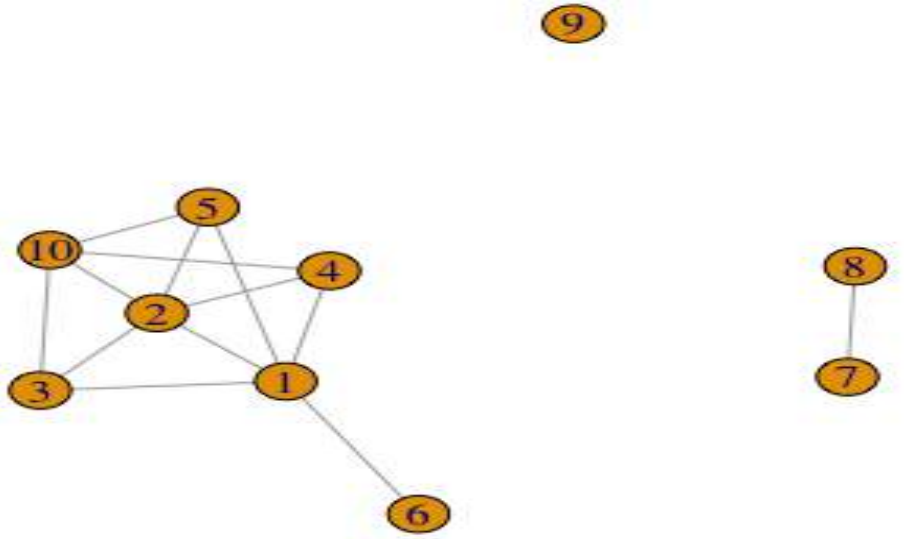
- If each and every vertex of one graph corresponds to each and every vertex of another graph, then both the graphs are said to be Isomorphic.



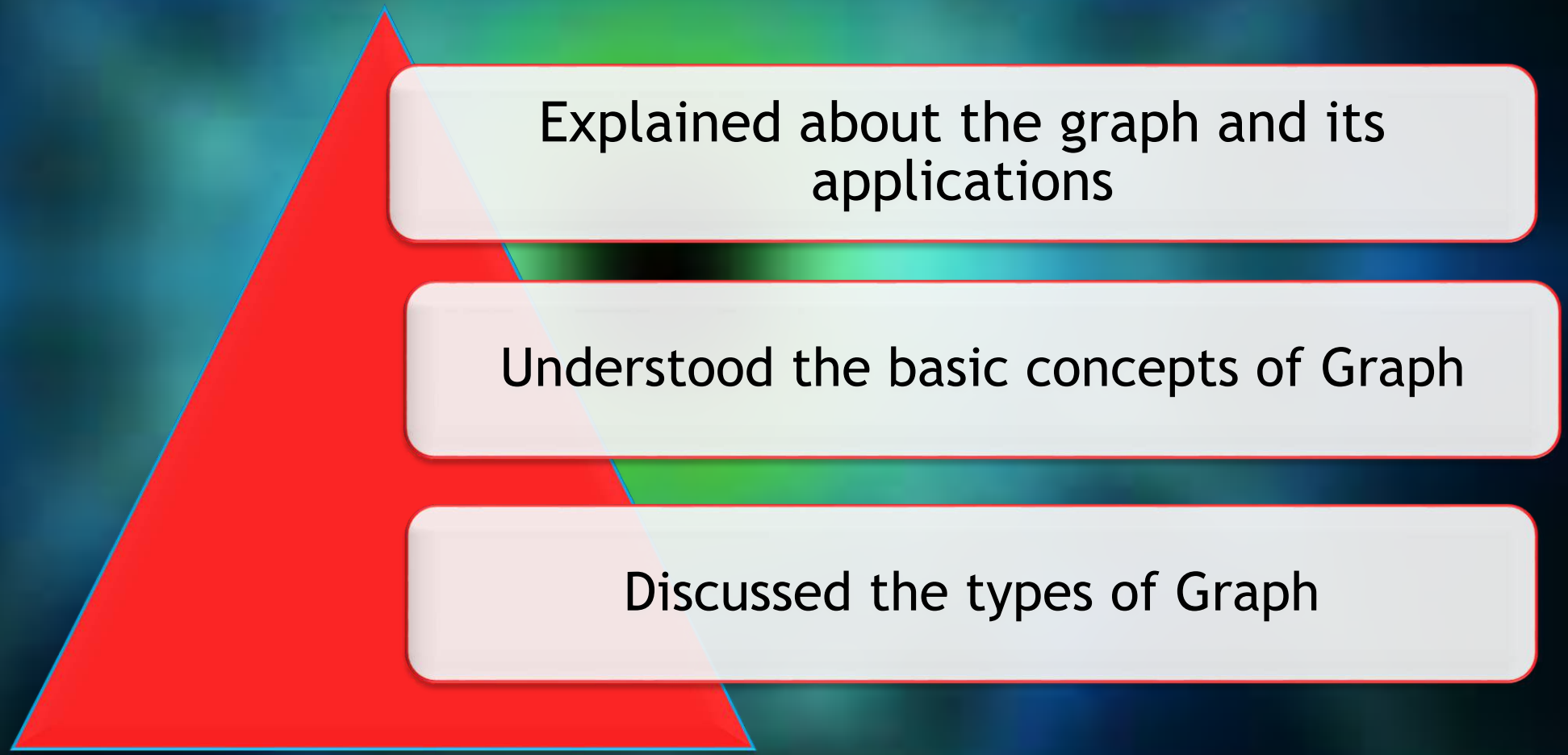


Disconnected Graph

- Disconnected graph has two or more than two connected graphs.
- Connected graphs of a disconnected graph are known as Components of Disconnected graph.
- Following example of disconnected graph consists of 3 components, i.e., 3 connected graphs; in which one is isolated graph and two are connected graphs.

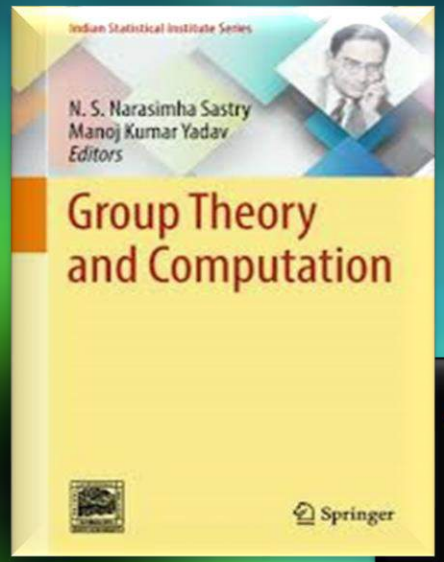


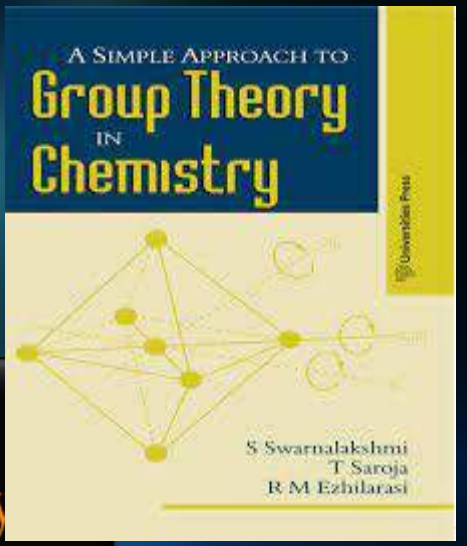
Summary

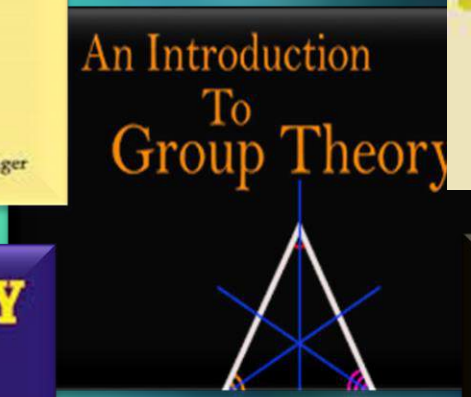

Explained about the graph and its applications
Understood the basic concepts of Graph
Discussed the types of Graph

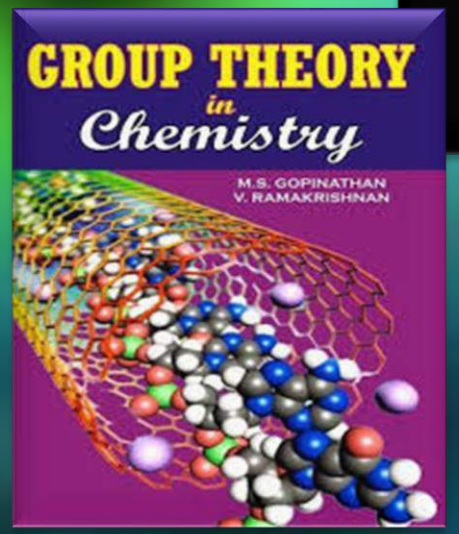
Reference

- https://medium.com/basecs/a-gentle-introduction-to-graph-theory-77969829ead
- https://www.britannica.com/topic/graph-theory
- https://towardsdatascience.com/what-is-graph-theory-and-why-should-you-care-28d6a715a5c2?gi=d25cf4837513









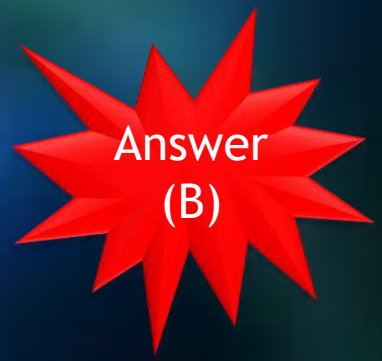
Self-Assesment

Q1. A Graph is a collection of….?

(A) Rows and Columns

(B) Vertices and edges

(C) Equations



Self-Assesment

Q2. The degree of any vertex of graph is ….?

A

The number of edges in a graph

B

The number of vertex in a graph

C

The number of vertices adjacent to that vertex

D

The number of edges incident with vertex

Answer (B)

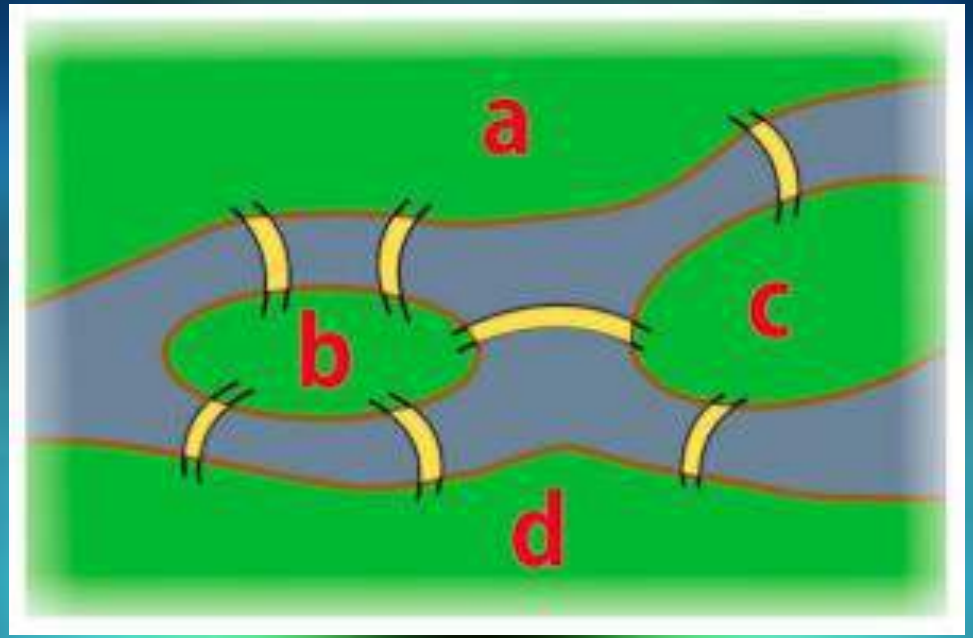
Self-Assesment

Q3. Suppose G and H are isomorphic graphs. If G is connected, then H is connected.

True

False

Answer
(True)



THANK YOU